Analysis of the Orbit Response Matrix Measurement for PSR

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1 Introduction

The orbit response matrix (ORM) matheod has been used to measure and calibrate the linear optics for many electron storage rings since J. Safranek's work.

It has been applied to uncover malfunctioning BPMs, the roll angle of magnets, mis-alignment of higher order multipoles, calibration of magnet power supplies, etc.

2 The Orbit Response Matrix Method

Definition of orbit response matrix

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N_b} \end{pmatrix} = \mathbf{M} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{N_k} \end{pmatrix} \tag{1}$$

Theoretically

$$M_{ij} = G(s_i, s_j) = \frac{\sqrt{\beta(s_i)\beta(s_j)}}{2\sin(\pi\nu)} \cos(\pi\nu - |\psi(s_i) - \psi(s_j)|), \tag{2}$$

where s_i and s_j are positions of the ith BPM and jth kicker.

The basic idea in acclerator modeling is to adjust all the model parameters until the difference between the model matrix and the measured matrix is minimized.

$$V_k = \frac{M_{model,ij} - M_{meas,ij}}{\sigma_i}, \tag{3}$$

where $k = i \times N_b + j$ is the index number,
Consider the calibration of BPM gain g_i and kicker angle calibration f_j

$$M_{meas,ij} = \frac{M_{data,ij}}{g_i f_j}. \tag{4}$$

$$\chi^2 = \sum_{i,j} V_k^2(i,j) \tag{5}$$

The problem is now to find Δx_n such that

$$V_k(\{x_n + \Delta x_n\}) \approx V_k(\{x_n\}) + \sum_n \frac{\partial V_k}{\partial x_n} \Delta x_n = 0, \tag{6}$$

has the best solution. This can be achived by solving linear equations with singular value decomposition(SVD) iteratively.

The computation of $\frac{\partial V_k}{\partial x_n}$ matrix

$$\frac{\partial V_k}{\partial x_n} = \frac{V_k(x_n + \Delta x_n) - V_k(x_n)}{\Delta x_n} \tag{7}$$

The V_k vector has $N_b \times N_k$ elements and there are N_p model parameters, and thus $\frac{\partial V_k}{\partial x_n}$ matrix will have $N_b \times N_k$ rows and $N_p + N_b + N_k$ columns. We should run MAD once for each of N_p model parameters.

Convergence condition

$$\sqrt{\sum_{n=1}^{N} \frac{1}{N} (\frac{x_{n,\mathrm{new}} - x_{n,\mathrm{old}}}{x_{n,\mathrm{old}}})^2} < C_1$$

3 Measurement and Data Analysis

PSR: circumference 90.26m, injection energy 800MeV, 10 super-period FODO cells

9 vertical steerers, 19 BPMs. The bump current are +4A and -4A, corresponding to the kick angle of about 1 mrad.

The χ^2 were brought down to 5.1 from 33.2 for +4A, 5.4 from 39.3 for -4A after fitting. If set σ_i to actual value 0.2mm, the actuall χ^2 becomes about 135. This corresponds to a χ^2 of 0.8 per data point.

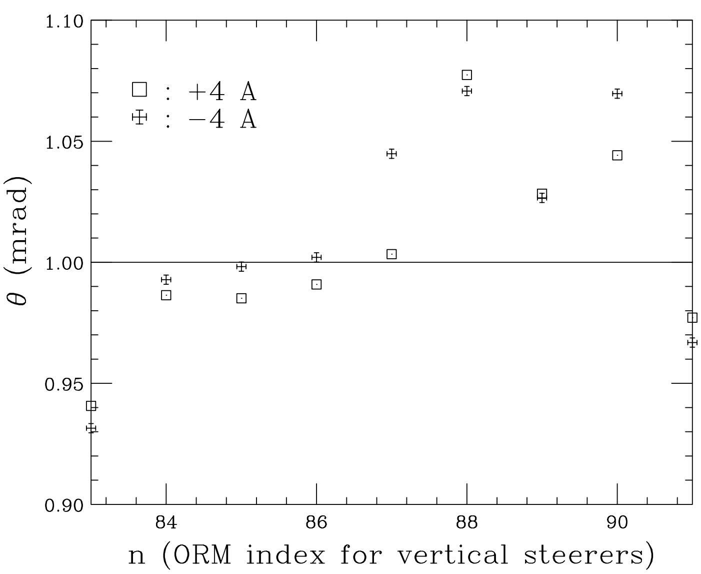

Figure 1: The vertical steerer's calibration in mrad for ±4 A of current. The steerer's calibration brought the χ^2 from 33. to 24.

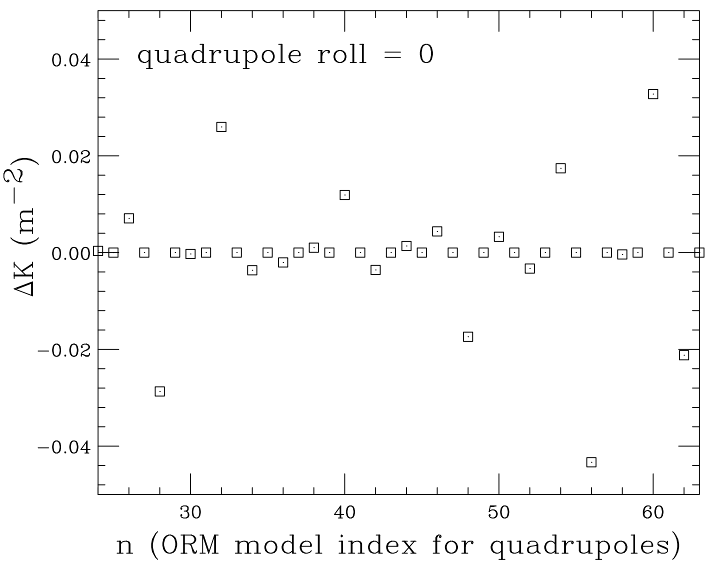

Figure 2: Quadrupole parameters.

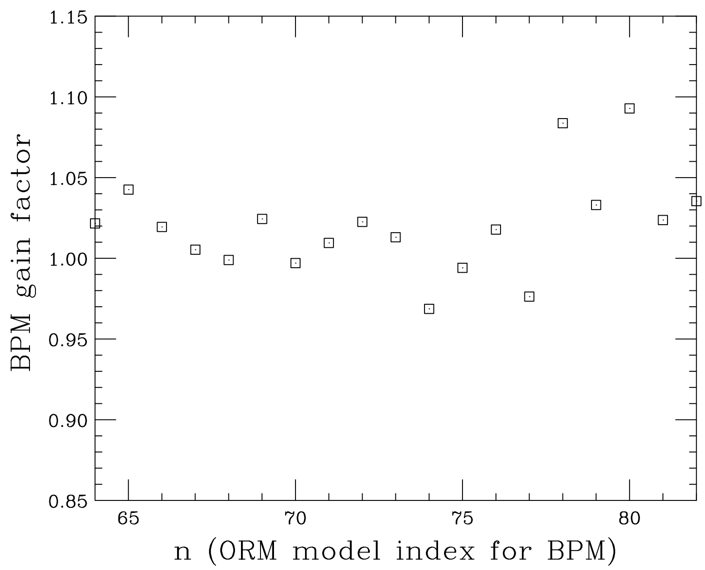

Figure 3: BPM gains. The actual BPM gains are 1.0 plus values shown in figure.